TEST METHODS AND RESULTS
FOR ADAPTIVE EQUALIZERS

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ABSTRACT
Multipath distortion has been a major source of data corruption in aeronautical telemetry signals for decades. In recent years, however, adaptive equalizers have begun to appear in telemetry receivers. These equalizers offer the promise of mitigating or even eliminating the damage done by the multipath channel, and many ranges are adopting their use. Unfortunately, there have not been any standardized tests by which to quantify the efficacy and limitations of adaptive equalizers. This paper presents a generalized test methodology for making a quantitative performance assessment of any adaptive equalizer, along with representative test results for one particular adaptive equalizer.

INTRODUCTION
Multipath propagation is a well-known nemesis of telemetry signals, so we will provide a brief overview of the phenomenon and put it on a mathematical foundation. Next, we will present a high-level generic overview of how adaptive equalizers operate. Finally, we will describe a test methodology and sample test results to show what the test results reveal about the equalizer.

THE MULTIPATH CHANNEL
The name “multipath” is pretty self-explanatory. As depicted in Figure 1, the signal transmitted from the test article reaches the receiving antenna by multiple propagation paths, each with different path lengths. Since the speed of RF signal propagation is finite, these signals arrive misaligned in time.

Figure 1 - Multipath Channel Geometry, from [1].
If we assume that the channel geometry does not change rapidly (compared to the bit rate), then we can write an expression for the impulse response of the channel as shown in Figure 2. While this expression can describe an arbitrary number of paths, most channel sounding tests have shown that two or three paths (direct path plus one or two reflections) generally provide a suitably complete representation of the propagation channel.

![Figure 2 - Multipath Channel Impulse Response, from [1].](image)

Given the time domain description of the multipath channel in Figure 2, we can derive the effective frequency response of the channel, which is helpful in understanding how the channel parameters such as differential delay and phase shift affect the telemetry signal. Figure 3 depicts the transfer function of the simplest possible multipath channel, the two-ray model.

![Figure 3 - Two-Ray Channel Frequency Response, from [1].](image)

Note three key features of the two-ray channel:

1. The channel response is periodic in frequency, with nulls occurring where the two rays align out of phase, at intervals that are inversely related to the differential delay. In other words, the longer the delay, the more spectral nulls that appear in any given bandwidth.
2. The depth of the spectral null is determined by the magnitude of the second ray (we assume that the direct path has magnitude one and phase zero).
3. The frequency at which the null in the channel occurs moves linearly with the phase of the second ray.

These three critical axes in the parameter space will be explored in our test procedure.
GENERIC EQUALIZERS

The use of adaptive equalizers to mitigate multipath dates back to at least the mid 1960s. See [2], for example. In the intervening decades, countless equalizer structures have been proposed and analyzed, as well as many algorithms for controlling their adaptation. Our present objective is not to review or comment on these equalizer designs, but rather to present a method for evaluating the performance benefit of any equalizer. For this purpose, the generic structure shown in Figure 4 will serve our needs. We will make only the most basic assumptions about this diagram, namely that

1. The information flowing through the equalizer is digital data, and
2. A reduction in the number of bit errors at the output is an improvement.

Under the assumption that the figure of merit for the equalizer should be the extent to which it reduces bit errors, now we “only” need the test equipment and a procedure to quantify this. First, let’s explore the qualitative behavior of multipath and an equalizer’s ability to mitigate it. Refer to Figure 5, which shows the eye pattern and spectrum for a PCM/FM signal that has been corrupted by two-ray multipath, both before and after equalization (left and right, respectively).

The frequency selective characteristic of the multipath channel is quite apparent in the lower left of Figure 5. As tabulated on the right, the differential delay is equal to one bit period (100 ns at 10 Mbps), the magnitude of the second ray is 0.9, and the phase shift is 150 degrees. (As before, we assume the direct path scale factor is 1+j0).

With the two-ray channel in this static condition, we can measure the BER both with and without the equalizer. This is the central principle of the proposed test procedure, but we still need to define some additional test conditions.

In an arbitrary multipath channel, each path is characterized by a delay, amplitude, and phase shift (all of which are potentially time-varying). There may be two, three, or more paths contributing. Furthermore, the impact of the multipath depends on modulation, bit rate, SNR, and

Figure 4 - Generic Adaptive Equalizer.

TEST PROCEDURE
more. Accounting for all possible configurations, we would need a 10-dimensional universe to plot the results. We have way too many test points, so we propose to simplify as follows.

While “real” multipath channels may have many reflections contributing to the overall transfer function, the parameters of a simple two-ray model can always be configured to stress the equalizer. Therefore, we propose to start with the basic two-ray model, which is easy to synthesize and still allows a range of channels from easy to impossible to equalize. Also, we propose to test at only one “meaningful” SNR. This should be high enough that the equalizer works on mitigating multipath, not rejecting noise, but not so high that there are never any bit errors, and it should reflect actual use cases. We propose to test at the RF level that yields 20 dB $E_b/N_0$ without the multipath channel response.

As far as the channel itself, we have previously observed that the static two-ray channel response depends on the delay, magnitude, and phase of the second ray. To create a range of channels that should cover the gamut from “piece of cake” to “holy cow”, we propose the following values:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delays (in bits)</td>
<td>0.5, 1, 2, 5, 10, 20, and 50</td>
<td>Delays much shorter than about 0.5 bit are essentially flat fades, where the signal power is simply gone. Equalization cannot help.</td>
</tr>
<tr>
<td>Amplitudes</td>
<td>0.5 to 0.9 in steps of 0.1</td>
<td>If this is too easy, include 0.95 and 0.98</td>
</tr>
<tr>
<td>Phases</td>
<td>0° to 360° in 10° steps</td>
<td>Phase determines the position of the spectral nulls within the signal bandwidth.</td>
</tr>
</tbody>
</table>

We have suggested that the figure of merit for adaptive equalizer performance should be the degree to which it lowers the bit error rate. While this is conceptually straightforward, experimentally it can lead to some very time-consuming tests. For this reason, we propose to
utilize the data quality metric (DQM), which has been introduced in telemetry receivers in recent years. DQM is essentially the negative of the log likelihood ratio (LLR), which is defined as

\[
    \text{LLR} = \log_{10} \left( \frac{P}{1-P} \right),
\]

where P is the probability of bit error.

In Appendix 2-G of IRIG 106-17, DQM is defined as a scaled version of the LLR, using a scale factor of \(-\left(\frac{2^{16}}{12}\right)\), yielding a 16-bit unsigned integer value \((0 \leq \text{DQM} \leq 65,535)\). For our present purposes, we will use the DQM value before the scaling, yielding a value between 0 and 12, which is (very nearly) the exponent of 10 when BER is expressed in scientific notation. See Figure 6 for some numerical examples.

<table>
<thead>
<tr>
<th>P</th>
<th>LR</th>
<th>DQM in Appendix 2-G</th>
<th>DQM in this paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.00000E+00</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>1.00E-01</td>
<td>1.11111E-01</td>
<td>5211</td>
<td>0.954</td>
</tr>
<tr>
<td>1.00E-02</td>
<td>1.01010E-02</td>
<td>10899</td>
<td>1.996</td>
</tr>
<tr>
<td>1.00E-03</td>
<td>1.00100E-03</td>
<td>16382</td>
<td>3.000</td>
</tr>
<tr>
<td>1.00E-04</td>
<td>1.00010E-04</td>
<td>21845</td>
<td>4.000</td>
</tr>
<tr>
<td>1.00E-05</td>
<td>1.00001E-05</td>
<td>27307</td>
<td>5.000</td>
</tr>
<tr>
<td>1.00E-06</td>
<td>1.00000E-06</td>
<td>32768</td>
<td>6.000</td>
</tr>
<tr>
<td>1.00E-07</td>
<td>1.00000E-07</td>
<td>38229</td>
<td>7.000</td>
</tr>
<tr>
<td>1.00E-08</td>
<td>1.00000E-08</td>
<td>43691</td>
<td>8.000</td>
</tr>
<tr>
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<td>1.00000E-09</td>
<td>49152</td>
<td>9.000</td>
</tr>
<tr>
<td>1.00E-10</td>
<td>1.00000E-10</td>
<td>54613</td>
<td>10.000</td>
</tr>
<tr>
<td>1.00E-11</td>
<td>1.00000E-11</td>
<td>60075</td>
<td>11.000</td>
</tr>
<tr>
<td>1.00E-12</td>
<td>1.00000E-12</td>
<td>65535</td>
<td>12.000</td>
</tr>
</tbody>
</table>

Figure 6 - DQM in IRIG 106 Compared to DQM in this Paper.

The benefits of using a properly calibrated DQM as the figure of merit for equalizer performance are two-fold:

1. DQM measurements can be collected much, much faster than BER measurements, dramatically reducing the test times.
2. DQM leads to an intuitively satisfying presentation of the test results, as described below.

As we have previously described, the phase of the second ray essentially defines the position of the spectral nulls along the frequency axis. Not surprisingly, the damage to the signal is a sensitive function of this parameter. To help gain insight into this characteristic, we propose to plot the measured results in polar form, using DQM plotted as the radius (bigger is better), and angle to represent the phase of the second ray, in the natural relationship. This leads to the following step-by-step procedure:
Before collecting equalizer performance data, we propose to compare the measured BER performance (collected with a conventional bit error rate test set) to the DQM reported by the receiver under test. A fixture for performing such calibration is shown in Figure 8.

For receivers that support the Data Quality Encapsulation (DQE) format (also defined in Appendix 2-G of IRIG 106-17), the calibration procedure is straightforward. First, synthesize the multipath-impaired RF signal. Allow the UUT to recover the “corrupted” data and output this data in DQE format. From the DQE format, extract the frame sync word, including the 16-bit DQM value. At the same time, measure BER of the payload data. Convert the measured BER to a log likelihood ratio as described above, and then compare DQM derived from the BER measurement to the DQM reported by the UUT. An example showing two different multipath amplitudes and 36 different multipath phases is depicted in Figure 9. Since this calibration procedure requires the actual measurement of BER, it can be very time consuming. However, once the DQM value reported by the UUT is known to be accurate, all the other tests can be run much faster, sometimes by several orders of magnitude.

**CALIBRATION**

As with any other test procedure, calibration of the test equipment is important. Therefore, before collecting equalizer performance data, we propose to compare the measured BER performance (collected with a conventional bit error rate test set) to the DQM reported by the receiver under test. A fixture for performing such calibration is shown in Figure 8.

For receivers that support the Data Quality Encapsulation (DQE) format (also defined in Appendix 2-G of IRIG 106-17), the calibration procedure is straightforward. First, synthesize the multipath-impaired RF signal. Allow the UUT to recover the “corrupted” data and output this data in DQE format. From the DQE format, extract the frame sync word, including the 16-bit DQM value. At the same time, measure BER of the payload data. Convert the measured BER to a log likelihood ratio as described above, and then compare DQM derived from the BER measurement to the DQM reported by the UUT. An example showing two different multipath amplitudes and 36 different multipath phases is depicted in Figure 9. Since this calibration procedure requires the actual measurement of BER, it can be very time consuming. However, once the DQM value reported by the UUT is known to be accurate, all the other tests can be run much faster, sometimes by several orders of magnitude.
It is worth noting here that the proposed test procedure can be performed on receivers that do not have accurate DQM, or any DQM at all. Simply measure the BER, convert it to DQM, and proceed as though the UUT reported an accurate DQM value. It’s also worth noting that both speed and accuracy of the tests can be improved if two copies of the UUT are available, so that “EQ off” and “EQ on” cases can be run simultaneously.

**STATIC CHANNEL TEST RESULTS**

We now have the tools and technique to characterize the equalizer benefit for a static two-ray channel. We proposed to capture the DQM from the UUT with the equalizer both off and on (or measure BER and compute DQM from the BER), and then plot DQM vs. delay path phase, in polar form, using DQM as the radius and phase of the delayed path as the angle. Graphically, the result will be a distorted “hoop”, where a larger radius (higher DQM) is better. Because the phase of the delayed path determines where the spectral nulls appear, some angles will be worse than others. To quantify the performance enhancement of the equalizer we need only to observe that the radius in these plots is (essentially) the logarithm of the BER. Therefore, if we compute the point-by-point difference $\text{DQMEQ-on} - \text{DQMEQ-off}$, we will directly get the number of orders of magnitude improvement in BER from using the equalizer. The EQ off, EQ on, EQ benefit plots are shown in Figure 10 for a sample case of 10 Mbps SOQPSK, with five different delays.

Several interesting observations can be extracted from Figure 10. First, we can see that when the delay is only a single bit period, there are “lucky” phases (approximately ±30-degree range around zero degrees) where the multipath does very little damage. Spectrally, this occurs when the multipath nulls “straddle” the main lobe of the signal. When the phase of the reflected path is 180 degrees, on the other hand, the spectral null lands right in the center of the signal spectrum, and the damage is severe. The first row of Figure 10 shows this case. It’s not presented here but testing with PCM/FM has shown that because the spectrum is roughly twice as wide as SOQPSK, there is no “sweet spot” where the multipath damage can be avoided.
Figure 10 - Equalizer Benefit Plots, 10 Mbps SOQPSK, Delays of 1, 2, 5, 10, and 20 bits.
It is also interesting to note that this particular equalizer does less well correcting multipath damage at approximately ±115 degrees than it does repairing even worse damage at 180 degrees.

Next, we can see that as the delay of the second path is increased, and the number of spectral nulls increases in proportion, the phase of the delayed path becomes less important. This is intuitively satisfying, since shifting the phase causes the nulls to sweep across the signal, but as one moves out of the spectrum, another one moves in.

We can also see that, as expected, increasing the amplitude of the second ray causes more damage to the signal. Finally, we see that for this particular equalizer, the benefit of using the equalizer increases as the delay actually increases – but only up to a point. When the delay reaches 20 bit periods, the benefit is starting to diminish. Recall that our objective was to find the limits of the equalizer’s “restorative powers”, so to speak, and we have done so.

**DYNAMIC CHANNEL TEST RESULTS**

The procedure described above yields a quantitative evaluation of the equalizer when the multipath channel is static, but all equalizers of interest in the aeronautical telemetry channel must be adaptive, because the channel geometry is continuously changing. Therefore, we also need a procedure for a dynamic channel. To achieve this, we propose to borrow from the “break frequency test” for diversity combiners as defined in IRIG 118, Procedure 5.7, which determines the fading frequency at which the combiner performance starts to degrade.

In a multipath channel, the amplitude, delay, and phase of the delayed path all change as the test article moves. However, the most pronounced effect of target motion is the variation in the phase of the reflected path, which causes the spectral nulls to sweep through the signal spectrum. Consequently, we propose to stress the equalizer by sweeping the nulls faster and faster, until the equalizer benefit starts to drop. This sweeping is achieved by simply “spinning” the phase of the second ray. We will then define the figure of merit as the “break frequency” of the adaptive equalizer as the spin rate at which the benefit is only half (in DQM improvement) as what it was in the static case.

Specifically, we propose to measure the BER averaged over all phases and convert this to DQM. This method will be slow. Alternatively, we can measure DQM directly, but must take care to average it correctly (see below). For consistency with the static plots, we plot DQM versus “spin rate”, with multiple delay path amplitudes on one chart, and separate charts for each delay value. An example is shown in Figure 11.

While the proposed procedure treats the phase of the second ray as an independent variable, it should be noted that phase change is fundamentally caused by the motion of the test article. Therefore, higher frequencies (C band, for example) will experience a greater rate of phase change than lower frequencies on the same aircraft, because the wavelength is shorter. This does not alter our proposed procedure; it simply means that a C-band signal will experience a higher spin rate than an L or S band signal emanating from the same moving aircraft.
While recording DQM directly yields enormous reductions on test time, averaging DQM for this test must be done correctly. Recall that DQM is the Log Likelihood Ratio. The logarithm is what gives it such a wide dynamic range, but it also means that averaging requires the “anti-log” function. For example, suppose five DQM values to be averaged are \{12, 12, 12, 12, 2\}. Simply summing these five values and dividing by 5 yields an incorrect DQM$_{avg}$ of 10.

The correct procedure is to convert DQM to probability using $P = 10^{-DQM} / (1 + 10^{-DQM})$, average the probabilities to get $P_{avg} = 0.001980198$, and then compute $-\log_{10} \left\{ P_{avg} / (1 – P_{avg}) \right\}$ to get $DQM_{avg} = 2.702$

**CONCLUSIONS**

We have presented a generalized test methodology for making a quantitative performance assessment of any adaptive equalizer, along with representative test results for one particular adaptive equalizer. We have also defined a calibration procedure for the test fixture used in the performance evaluation. The procedure leverages DQM and DQE as defined in IRIG 106-17, which results in rapid, accurate, and meaningful test results, and makes it possible to compare multiple equalizers on a common scoring system.

**ACKNOWLEDGEMENTS**

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**REFERENCES**
